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Translated by M.D.F.

PMM U.S.S.R.,Vol.49,No.1,pp.120-122,1985
Printed in Great Britain

0021-8928/85 $10.00+0.00

EVOLUTION OF A WEAK SIGNAL IN A MAGNETIC MATERIAL*

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The variation of a weak arbitrary perturbation of the magnetic field with time in a magnetic material is investigated. It is assumed that the cubic relation connecting the strength H of the magnetic field with its induction B holds for the material in question. Such a relationship represents a special case of the relation between these quantities for the initial magnetization of the majority of magnetic materials /1/. It is shown that after a certain finite time the signal degenerates into a simple wave and a closing trailing shock. The profile area remains constant and equal to the area of the initial signal.

The formation of shock waves at the front and the slope of the electromagnetic waves was studied earlier in /2, 3/ for a magnet magnetized to saturation, and for the case of precession of the constant magnetization vector.

Let a weak signal $B(x, t_0)$, be generated in the magnetic material at the instant t_0, moving over zero background in the positive direction of the x axis (Fig.1). We shall consider the variation in the given profile with time for the case when the relation connecting H and B has the form

$$H = a_0 (B + B^*) + \gamma (B - B_0)^3 \tag{1}$$

Here a_0, γ are certain constants and $a_0 \gg \gamma B_0^2$; B_0 is the value of the magnetic induction for which $d^2H/dB^2 = 0$; B^* is chosen so that when $B = 0$, H is also zero.

The evolution of a weak signal in a magnetic material was investigated in /4/ for the case of a quadratic dependence of H on B.

When the relation $H(B)$ is given, the velocity of small perturbations $u = c\sqrt{a_0}$ is nearly equal to the velocity of the resulting discontinuities /5/

$$v_p = c\{a_0 + \gamma[(B - B_0)^2 + (B - B_0)(B_1 - B_0) + (B_1 - B_0)^2]\}^{1/2}$$

Therefore the perturbations reflected from the discontinuity can be neglected and the initially specified signal will propagate in one direction (e.g. to the right) in the form of a simple wave, with the signal area preserved.

$$s = \int_{x_1}^{x_2} B\, dx$$

Since the rate of propagation of simple waves $a = c\sqrt{dH/dB}$ depends on B, different points of the profile will move through the magnetic material with different velocities, and this will lead to signal distortion, non-uniqueness of the solution, and the formation of discontinuities.

When the signal profile is deformed, the area s_1 corresponding to the extension of the shock zone will be equal to the area s_2 which would be traversed over the same time by a simple wave in the case of non-uniqueness. Indeed,

$$s_1 = v(B - B_1) = c\left(\frac{H - H_1}{B - B_1}\right)^{1/2}(B - B_1) =$$

$$c\left[\frac{a_0(B - B_1) + \gamma(B - B_0)^3 - \gamma(B - B_1)^3}{B - B_1}\right]^{1/2}(B - B_1) \approx$$

$$ca_0^{1/2} + \frac{\gamma c}{2a_0^{1/2}}\left[(B - B_0)^3 - (B_1 - B_0)^3 \approx\right.$$

$$s_2 = \int_{B_1}^{B_2} a\, dB = \int_{B_1}^{B_2} c\left(\frac{dH}{dB}\right)^{1/2} = c\int_{B_1}^{B_2} [a_0 + 3\gamma(B - B_0)^2]^{1/2}\, dB$$

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*Prikl.Matem.Mekhan.,49,1,161-163,1985

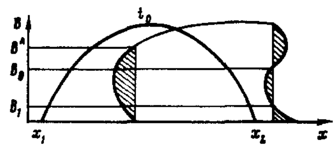

Fig.1

Fig.2

As time elapse, the solution takes the following form: discontinuities form in front and at the back, and a simple wave propagates between them. We shall assume that the width of the initial impulse is small compared with the characteristic dimensions of the magnetic material. In this case the simple wave can be assumed to be centered.

Since the solution consists of simple, plane polarized waves and discontinuities, the magnetic field will remain parallel to a certain direction, e.g. $\mathbf{B} = B\mathbf{e}_y$, and the electric field will be perpendicular to the magnetic material $\mathbf{E} = E\mathbf{e}_z$.

Maxwell's equations for the simple centered wave yield

$$B = B_0 + \frac{1}{c}\left(\frac{\xi^2 - a_0 c^2}{3\gamma}\right)^{1/2}, \quad \xi = \frac{x}{t} \tag{2}$$

We shall assume that in the initially specified profile $B_{max} > B^{\wedge}$, where $B^{\wedge}$ is the value of the magnetic induction for which the following relation holds:

$$\left.\frac{dH}{dB}\right|_{B^{\wedge}} = \frac{H(B^{\wedge})}{B^{\wedge}} \tag{3}$$

In this case the rear discontinuity will be evolutionary /5/ and the magnetic field will vary at the discontinuity from $B^{\wedge}$ to zero. From (3) we obtain $B = {}^3/_2 B_0$.

The front discontinuity in which the field increases from zero to some value B, will be weakened by the action of the centered wave arriving from behind. When the magnetic induction behind the discontinuity falls to the value B^{**}, where $v(B^{**}, 0) = a(0)$, then the discontinuity ceases to be evolutionary and decomposes into a discontinuity and a simple wave enveloping it from the front (Fig.1).

The condition that, for a value of the magnetic induction B_1 (at the point B_1 the simple wave becomes a discontinuity), the velocity of the simple wave is equal to the velocity of the discontinuity yields the value of the magnetic field B_1 behind the discontinuity

$$B - B_0 = 2(B - B_1) \tag{4}$$

According to (4) the rate of propagation of the front discontinuity can be found from the condition

$$v_p = \frac{dx}{dt} = c\left(\frac{H - H_1}{B - B_1}\right)^{1/2} = a(B_1) \tag{5}$$

Since

$$v_p/c = \{a_0 + \gamma[(B - B_0)^2 + (B - B_0)(B_1 - B_0) + (B_1 - B_0)^2]\} = [a_0 + {}^3/_4\gamma(B - B_0)]^{1/2} \approx a_0^{1/2} - 3\gamma a_0^{-1/2} \tag{6}$$

therefore

$$v_p/c - a_0^{1/2} = {}^1/_4[a(B)/c - a_0^{1/2}]$$

(the velocity of the centered wave at the point B: $a(B) = c(dH/dB)^{1/2} = \xi$).

Taking into account relation (6) which holds for (1) when $a_0 \gg \gamma B_0^2$, we write the differential equation (5) in the form

$$\frac{dx}{dt} = v_p = \frac{1}{4}\left(\frac{x}{t} - ca_0^{1/2}\right) + ca_0^{1/2}$$

Using the formulas $x' = x - ca_0^{1/2}t$; $t' = t$ to change to a coordinate system moving with constant velocity $ca_0^{1/2}$, with respect to the system used, we obtain

$$\frac{dx'}{dt'} = \frac{1}{4}\frac{x'}{t'} \tag{7}$$

Let us integrate (7) from some values x_0', t_0' to the running values $x_1' t'$ (the primes will be omitted from now on). The values x_0, t_0 can be found from the given form of the initial profile (after the rear and front discontinuity have merged)

$$x = At^{1/4}; \quad A = x_0 t_0^{-1/4} \tag{8}$$

Equation (8) represents the equation of the line of discontinuity in the xt plane. The line intersects the family of characteristics of the centred wave and intersects at

$x = (8a_0^{1/2} A^{4/3} \gamma B_0^2 c)^{1/3}$ the extreme left characteristic corresponding to the rear shock front. The front and rear discontinuities will merge. We can find from formula (8) that the time t in which the rear discontinuity will overtake the front discontinuity is given by the formula

$$t = \left(\frac{8a^{1/2} A}{3\gamma c B_0^2}\right)^{4/3} \tag{9}$$

Thus, when the dependence of H on B is given by (1), the initial signal $B(x, t) \subset B_{max} > B^{\hat{}}$, propagating on a zero background in a magnetic material is deformed into two shock fronts connected by a simple centred' wave, with the signal area remaining unchanged. The leading front begins to weaken with time due to the emergence of a simple wave before it, with the centred wave coming up from behind. When the time t given by (9) has elapsed, the back shock catches up with the front one and the initial signal deforms into a simple wave and a closing shock propagating along a zero backgroud and changing the value of the field to $B = {}^3/_4 B_0$. The magnitude of the shock gradually decreases as the simple wave spreads out, with the area of the signal remaining constant (Fig.2).

Since the dependence (1) of H on B represents a special case describing the initial segment of the magnetization curve of most magnetic materials /1/, it follows that the analytic solution obtained reflects qualitatively the pattern of the evolution of a weak signal during the initial magnetization.

The conclusions drawn here concerning the nature of the signal change are in good agreement with the experimental study of the formation of shock waves when an electromagnetic pulse propagtes through a magnetic material, when the material is remagnetized with reverse polarity along the hysteresis loop /6/.

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Translated by L.K.